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ABSTRACT

This paper summarizes the recent work of the Maryland Physics Education Research Group on analyzing student learning in introductory university physics. This work is carried out in a model of student thinking and learning based on principles from cognitive science. The model permits a better understanding of, and diagnosis for, the state of both individual students and classes. For the study and teaching of university physics, the research summarized here indicates that the context that determines a particular student response includes not only the question or problem and the classroom environment, but also unobservable (and uncontrollable) elements of the student's internal state. (Contains 17 notes.) (WRM)

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# Diagnosing student problems using the results and methods of Physics Education Research

Edward F. Redish
Department of Physics
University of Maryland,
College Park MD 20742-4111

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# Diagnosing student problems using the results and methods of Physics Education Research

Edward F. Redish
Department of Physics, University of Maryland
College Park, MD 20742-4111 USA

## Learning About Student Learning

In order to understand what is happening in our classrooms, we have to study our students and their responses to our instruction. Physics Education Research (PER) is the subject in which we study how students understand (and fail to understand) physics in order to help individual students get over their difficulties in leaning physics develop curriculum and materials that are more effective for students.

Physics education researchers have studied student difficulties with many subjects[1]

- Kinematics
- Dynamics
- Energy
- Heat and Temperature
- Pressure
- Mechanical waves
- Oscillations
- Electric fields
- DC circuits
- EM Fields
- Geometrical optics
- Physical optics
- Relativity
- Modern physics
- Quantum mechanics

This is an impressive list, and much has been learned about the problems students have in learning particular topics in physics. Is this sufficient? The list of topics studied sounds like the table of contents of a standard text. But we want our students to get more out of a physics class than just "the physics". We want them to develop scientific skills and ways of thinking. We'd like them to – at least in part – learn to "think like a physicist".

### Goals Beyond Content

In trying to convince students to take physics (and other departments to require that their students take physics) we often cite the many skills that can be developed listed as the result of studying physics.

- Learning how to solve complex problems
- Learning how to think scientifically
- Learning how to learn
- Developing an intuition for the physical world.

Yet we rarely evaluate our success in achieving these goals. How do we know the extent to which we are succeeding? How can we diagnose the state of an individual student or of a class? Most of our evaluations focus on content and miss these broader goals. If my students memorize lots of problems and can replay anything they've seen before but can't do any problem they haven't seen — no matter how similar…? If my students can solve a problem mathematically but can't tell me what the problem is

about, or what the answer means…? Every observant physics teacher has seen these sorts of results. Research evidence has now become very compelling to show that learning to solve algorithmic problems – even reasonably complex ones – does not imply that our students have developed a sound physical intuition or an understanding of even the simplest implications of what they have learned.

Eric Mazur cites a compelling example in his work with his students at Harvard University.[2] On an examination to his algebra-based physics class, he gave the problems shown in figure 1.

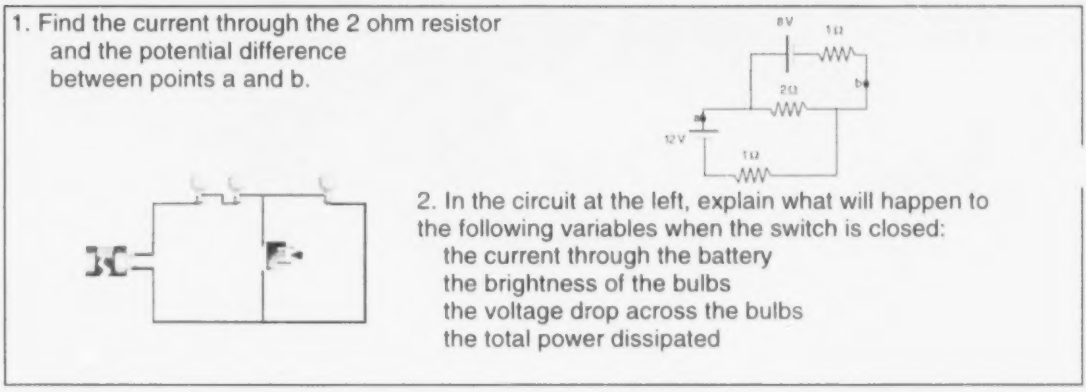
1. Find the current through the 2 ohm resistor
and the potential difference
between points a and b.
8V
1Ω
2Ω
a
b
12V
1Ω
2. In the circuit at the left, explain what will happen to
the following variables when the switch is closed:
the current through the battery
the brightness of the bulbs
the voltage drop across the bulbs
the total power dissipated

*Fig. 1: A quantitative and a qualitative problem in DC circuits (from Mazur, ref.2)*

The average score on the first problem was 75%. The average score on the second was 40%. Students found problem 2 much more difficult than problem 1, despite the fact that most physicists would consider the analysis of the second problem, the short circuit, much simpler; indeed, parts of it might be considered trivial. This study and many others[3] show that students frequently can solve complex algorithmic problems without having a good understanding of the physics.

The apparent inconsistency in what students appear to do suggests we have to think more carefully about what we are trying to accomplish. If we want to understand what elements to look for and what to try to evaluate we need to understand something about how students think. To do this we need to have some model of thinking and learning.

Often, faculty behave as if they think that a student's thinking is simply a storage box – a box into which we can put knowledge; that we can put knowledge into a student by simply presenting it to them and that all we have to know is whether the knowledge is "in there" or not. But cognitive and neuroscientists have been documenting for decades that the situation is considerably more complicated than that.[4]

## Models of Memory

Modern cognitive science now has complex and detailed structural information about how memory works. In some cases, the process is understood down to the level of neurons. We won't need all that. A few simple principles can help us understand some critical issues. The important ideas are

1. Memory has two components that our important for our consideration: working (or short term) memory and long-term memory.
2. Long-term memory contains facts, data, and rules for how to use and process them (declarative and procedural memory).
3. Recall from long-term memory is productive and context dependent.
4. Long-term memory is structured and associative.

In order to make these ideas clear, let's consider an example from student learning of university-level physics. An interesting example of productive reasoning appears in a recent paper from Lillian McDermott's group at the University of Washington.[5] The question shown in figure 3 was shown to a class of engineering physics students before and after instruction. *(Since the interference arises from the waves from the two slits interfering with each other, the pattern would go away and be replaced by an almost uniform brightness.)*

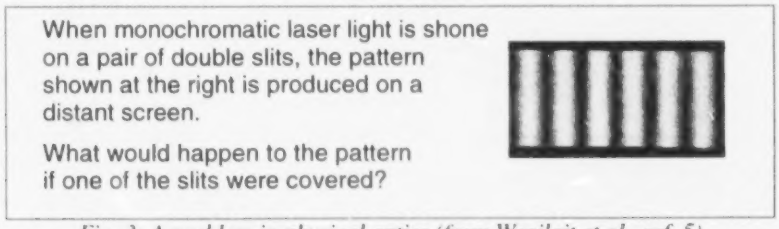
When monochromatic laser light is shone on a pair of double slits, the pattern shown at the right is produced on a distant screen.
What would happen to the pattern if one of the slits were covered?

*Fig. 3: A problem in physical optics (from Wosilait et al., ref. 5)*

More than half of the students expected part of the pattern would remain. Some said the left half of the lines would remain. Some said every other line would remain. None of the students had actually seen this happen, but they created their response from applying some general primitives (reduced cause leads to a reduced effect), perhaps supported by their experience with light and shadow.

The second important idea is that the knowledge in long-term memory is organized by patterns of association – probabilities that bringing one bit of knowledge out to working memory will lead to another bit of relevant knowledge. The fact that some bit of knowledge or know-how is "in there" doesn't help much if it doesn't come up when you need it. What's important is not just what knowledge you have but its functionality --how appropriately you access it and how well you can use it.

The context dependence of the cognitive response has implications for what students do (and how we interpret it). Students may hold contradictory models of physical situations at the same time. Problems seen as "equivalent" by an expert may not be so seen by a novice. Particularly cues in a problem may lead them to bring up one model or another.

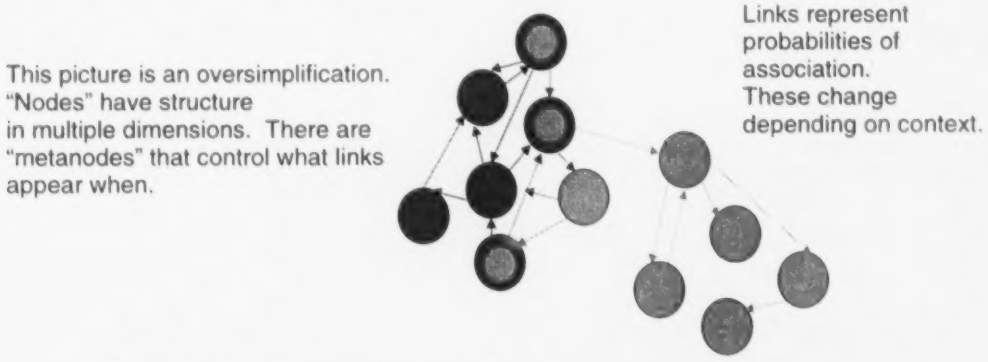
This picture is an oversimplification. "Nodes" have structure in multiple dimensions. There are "metanodes" that control what links appear when.
Links represent probabilities of association. These change depending on context.

*Fig. 4: A schematic geometrical representation of a pattern of association (schemas) in a knowledge structure.*

Sometimes, individuals may have a particularly robust pattern or cluster of associations – when one item is brought up in a particular type of context, other elements are likely to appear as well. In this case, we refer to the cluster of knowledge as a *schema*.[6] This is illustrated in the figure above.

Examples of schema and their context dependence can be found in Mel Sabella's thesis.[7] In one example, Sabella gave a problem in introductory mechanics in an interview to students taking a graduate class in the University of Maryland. The problem was chosen to require a mix of arguments using dynamics with forces together with work-energy arguments. Some of the students could do problems with forces or problems with energy but had serious difficulties in the context of the given problem. They were unable to make the links from their force schema to their energy schema. Figure 5 shows a schematic breakdown of the reasoning done by two of these students, one of whom had a well-integrated schema of forces and energy, and of a second who was unable to make connections between the two. He occasionally brought up the idea of energy, but in each case, the link was a "dead-end", with no connections to his knowledge of energy (at least while he was in the context of the particular problem presented.)

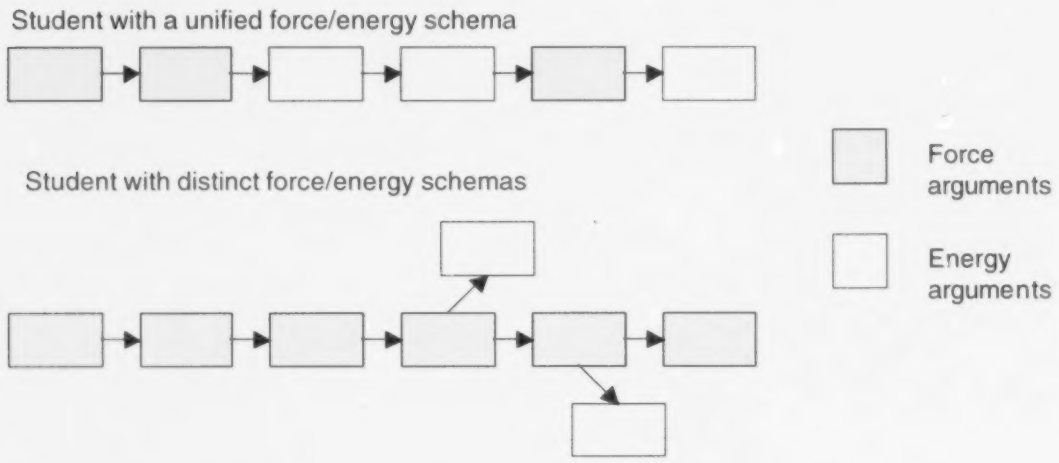


*Fig. 5: Pattern of reasoning of two students in response to a combined force/energy problem illustrating use of schemas.*

Our cognitive considerations suggest that in addition to having students master the physics content, we also want to consider

- the extent to which students have a conceptual understanding of the physics (the extent to which they see the physics as "making sense")
- the "robustness" of their knowledge (their ability to access the correct knowledge and use it appropriately)
- the way the students organize their knowledge (create robust "mental models" of physical systems that allow them to reconstruct the knowledge they need when they need it using links to appropriate productive reasoning).

## An Example: Mechanical waves and sound

In order to see how these ideas apply and effect curriculum development, let's consider an example: student understanding of mechanical waves and sound. Waves have the characteristic that the medium "recreates" and passes the disturbance onward. This leads to some interesting and counterintuitive properties, such as the fact that for linear media such as small amplitude deformations of strings, springs, and solids, sound, and electromagnetic fields, the speed of propagation is a characteristic of the medium,

not of the "intruding" disturbance. In these systems, the speed of propagation does not depend on the shape, amplitude, or frequency of the initiating disturbance.

Michal Wittmann's research[8] revealed that many students in our introductory calculus-based engineering physics class (18-19 years old) often had a serious misunderstanding of the phenomena. In a series of interviews, Wittmannn asked the students how a dust particle floating in the air in front of a loudspeaker would move when the speaker played a tone of constant frequency (low – about 10 Hz). A typical response is given below.

> *It would move away from the speaker, pushed by the sound wave ... I mean, sound waves spread through the air, ... the air is actually moving, so the dust particle should be moving with that air which is spreading away from the speaker.*
>
> *The sound wave "hits the particle with ... force. (drew figure shown at right)*

Since the sound wave is a pressure oscillation, the particle would oscillate back and forth. This, and many other students interviewed felt the particle would be driven away from the speaker.

This response, and the response to many other interviews and open-ended exam questions, students showed a common response. They used a *particle-pulse model* – a guiding analogy (a pattern of association) that a wave crest is like a Newtonian "point particle." The rest of the wave is ignored.

Wittmann's interesting observation was that in a multi-question diagnostic quiz on wave issues, almost all students in engineering physics used this analogy in some contexts and the correct reasoning in others. These observations are consistent with our schema model and imply that students do not "have the right answer or the wrong answer", but appear to have both answers at the same time.

## Student difficulties are not much affected by traditional instruction

It is well documented in the research literature[1] that student difficulties with physics are often quite difficult to change. We see the same thing here. In our traditional instruction, in each of 14 weeks, students are lectured to for 3 hours in classes of 100-200, solve about 10 "end-of-chapter" homework problems, spend one hour in a small class recitation with a teaching assistant going over those problems (mostly in a lecture-like format), and spend 3 hours in a traditional (rather cookbook) laboratory. In a preliminary investigation, after traditional instruction only about 25% of the students correctly described the dust particle as oscillating. Nearly half described it as being pushed away from the speaker.

### Curriculum to Address Difficulties with Sound

We modified the traditional class by replacing the recitation by a group-learning session with guided discovery worksheets focused on qualitative reasoning.[9] These tutorials use a cognitive-conflict model to induce conceptual change and require students to make predictions of what they expect to happen before they look at data or carry out qualitative experiments. The teaching assistants are trained and serve as facilitators, guiding each group's work by occasional careful questions. We created two worksheet tutorials using videos my students Bao Lei and Mel Sabella made at Dickinson College. In these lessons, students are asked to account for the motion of a flame observed in the video flickering in front of a loudspeaker. Students must resolve discrepancies between their descriptions and their observations and develop representations based on their observations of the videos. Two other wave phenomenon tutorials were also created and addressed the issues of the particle-pulse model.

## Evaluation: Pre- and Post-Instruction Wave Diagnostic Test

Our cognitive model and our observations of student response to waves questions strongly suggests that students do not have a "true value" of their knowledge of a particular subject. Given a large number of questions "seen as equivalent by experts", an individual student may respond using a variety of models. An exam question designed with "cues" or "triggers" to "see if the student really knows it" may give a positive response satisfying to both the student and instructor, but may give little information about when the students can use their knowledge.

In order to evaluate the student learning, we developed a variety of questions that offered students opportunities to use the particle-pulse guiding analogy. In the different contexts provided, almost all students mixed their use of the correct model with the popular (but incorrect) particle-pulse model. We measured success not only by the shift of the mean, but by the shift of the distribution.

A taut string is attached to a distant wall. A demonstrator moves her hand to create a pulse traveling toward the wall (see diagram). The demonstrator wants to produce a pulse that takes a longer time to reach the wall. Which of the actions *a-k* **taken by itself** will produce this result? More than one answer may be correct. If so, give them all. Explain your reasoning.

a. Move her hand more quickly (but still only up and down once by the same amount).
b. Move her hand more slowly (but still only up and down once by the same amount).
c. Move her hand a larger distance but up and down in the same amount of time.
d. Move her hand a smaller distance but up and down in the same amount of time.
e. Use a heavier string of the same length, under the same tension
f. Use a lighter string of the same length, under the same tension
g. Use a string of the same density, but decrease the tension.
h. Use a string of the same density, but increase the tension.
i. Put more force into the wave.
j. Put less force into the wave.
k. None of the above answers will cause the desired effect.

*Fig. 6: A multiple-choice multiple-response problem in mechanical waves.*

One example is given in figure 6. This is in a multiple-choice multiple response format. The students are instructed to give all the answer that are true. This question specifically demonstrates the students' use of mixed models since many give both responses in which the speed is determined by the medium and by

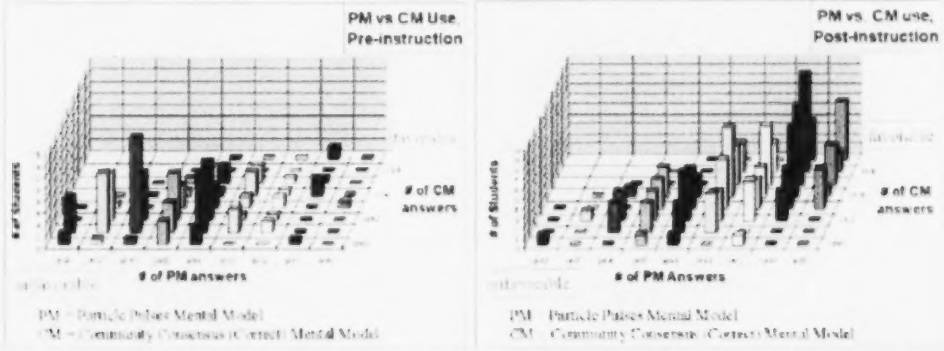
PM vs CM Use, Pre-instruction
# of Students
# of CM answers
# of PM answers
PM vs. CM use, Post-instruction
# of Students
# of CM answers
# of PM Answers

*Fig. 7: Distribution of pre-instruction and post-instruction responses on the wave diagnostic test showing number of questions answered by each student using the particle-pulse and correct models.*

the hand. They do not appear to notice the contradiction implied by having two ways of determining the speed that give different results. (In traditional classes after instruction nearly 75% of the students answered this question with both types of responses.

## Probing the mixing of student models with multiple choice tests

Traditional standardized exams in the US are set up as multiple choice in order to allow large numbers of students to be tested. These are usually simply scored by class averages of right and wrong. This analysis is based on the Spearman assumption (1904) at students have a "true" value, T, and that an examination yields a measured value, M, which is the true plus a random variable[10]

$$M = T + X \ .$$

### An Alternative Model

The cognitive theory we discussed above suggests that the student has no "true" value, but can most appropriately be considered as being in a mixed state. Since the one physical situation in which an object can exist in multiple states at the same time is in quantum physics, we use a quantum metaphor in developing new mathematical tools. In our proposed model, a student's state

$$|u\rangle = a_1|e_1\rangle + a_2|e_2\rangle + |X\rangle$$

may be mixed plus random.[11]

The evaluation is based on using the traditional hybrid model of test development. (See for example the description by Halloun and Hestenes of how they developed the predecessor of the Force Concept Inventory, ref. 3.) The process is as follows.

- Conduct qualitative research to identify student models underlying their spontaneous reasoning for a particular content area.
- Develop a theoretical framework to model the student reasoning process for that particular topic.
- Develop model-based multiple-choice exams to analyze student model use with quantitative measures.
- Use the results – including the student selection of wrong answers – from the quantitative research to facilitate the design of new instructions as well as new diagnostic and evaluation tools.

### Mathematical Representations

In many research studies (see ref. 1), it is found that the large majority of student answers can be described in terms of a small number of reasoning patterns. These usually include the correct (community consensus) model, one or more moderately consistent (in some sense) alternative[12] model, and a pattern of random and inconsistent answers. Bao Lei, a student from Nanjing whom I met at the 3rd US/China/Japan Conference on physics education, has recently completed a doctoral dissertation with me at Maryland. In it, he developed a mathematical structure using the quantum metaphor that provides a number of useful developmental and evaluation tools. Here I provide only a few examples. A more complete description is given in his dissertation.[13]

Assuming for simplicity one alternative student model, we introduce a "model space" with basis vectors

$$|e_1\rangle = (1,0,0)^T \quad |e_2\rangle = (0,1,0)^T \quad |e_3\rangle = (0,0,1)^T$$

We take these to be orthogonal and let $|e_1\rangle$ correspond to the correct model, $|e_2\rangle$ to an alternative student model, and $|e_3\rangle$ correspond to the student responding randomly (guessing or cueing on irrelevant elements). The state of the student is described, in analogy with a quantum amplitude, by the vector

$$|u_k\rangle = (\sqrt{q_1}, \sqrt{q_2}, \sqrt{q_3})^T$$

$$\langle u_k | u_k \rangle = \sum_{i=1}^{3} q_i = 1$$

where $q_i$ is the probability that the student will respond to a relevant problem using the $i$-th model.

### Model Analysis of a Multiple-Choice Test

As an example of the process, let us apply the method to a multiple-choice test developed using the hybrid model described above, the Force-Concept Inventory (FCI).[14] Five of the questions on the FCI address the student alternative conception known as the *Force-Motion model* (Questions 5, 9, 18, 22, 28). We choose the vectors $|e_1\rangle$, $|e_2\rangle$, and $|e_3\rangle$ to represent the following models:

- *Model 1*. A force is needed to maintain motion. (*incorrect*)
- *Model 2*. An unbalanced force produces a change in the velocity. No unbalanced force is needed to maintain a constant velocity. (*correct*)
- *Model 3*. Other ideas and random guessing. (*null model*)

To each question, we assign the responses that we expect are associated with a particular model. (This expectation needs to be confirmed by detailed interviews in which students consider the test items and explain their reasoning.) For the FCI, we get the following assignments.[15]

| *Question* | *Model 1* | *Model 2* | *Model 3* |
|---|---|---|---|
| **5** | a, b, c, | d | e |
| **9** | b, c | a, d | e |
| **18** | a, e | b | c, d |
| **22** | b, c, e | a, d | |
| **28** | a, d, e | c | b |

In a class of N students, the response of the k-th student can be described by the *student response vector*[16]

$$\vec{r}_k = (n_1, n_2, n_3)^T \qquad n_1 + n_2 + n_3 = m \qquad k = 1, \ldots N$$

where $n_1$ is the number of items the student answers using model 1, etc. If we interpret $n_1/m$ as the probability that the student will use model 1 on this test, then we can write the student state vector as

$$|u_k\rangle = \frac{1}{\sqrt{m}} (\sqrt{n_1}, \sqrt{n_2}, \sqrt{n_3})^T$$

Notice that the student state vector is *test-dependent*. That is, the student state vector represents an interaction of the student and the test. The student state vector should not be interpreted as an absolute description of some internal property of the student. Since it depends on the test as well as the student, the instructor using this method must first decide, based on his or her best judgment of appropriate goals and the learning situation, what it is that he or she wants to measure about student learning. As in quantum physics, the measurement affects the observation.

## The Density Matrix

The quantum metaphor suggests we introduce an additional tool to describe whether the student is in a pure or a mixed state: the *density matrix*. For each student we construct the matrix

$$\boldsymbol{D}_k = |u_k\rangle\langle u_k| = \vec{u}_k^{\,T} \otimes \vec{u}_k = \frac{1}{m}\begin{vmatrix} n_1 & \sqrt{n_1 n_2} & \sqrt{n_1 n_3} \\ \sqrt{n_1 n_2} & n_2 & \sqrt{n_2 n_3} \\ \sqrt{n_1 n_3} & \sqrt{n_2 n_3} & n_3 \end{vmatrix}$$

As it stands, the density matrix for an individual student contains no more information that the student scores, $n_1$, $n_2$, and $n_3$. However, when the density matrices for all the students in a class are added, this situation changes.

We construct the density matrix of a class by summing over the individual student density matrices:

$$\boldsymbol{D} = \frac{1}{N}\sum_{k=1}^{N} \boldsymbol{D}_k$$

If we only summed the individual scores rather than the density matrices, we would lose information about how many students are confused (i.e., are in mixed model states). For example, consider the class density matrices shown below. In the first case, all the students have the same consistent model (not necessarily the correct one). In the second case, half of the students consistently have model 1, 30% consistently have model two, and the remaining 20% are guessing randomly. In the third example, a number of students have mixed models. This is revealed by the presence of off-diagonal elements.

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \qquad \begin{vmatrix} 0.5 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.2 \end{vmatrix} \qquad \begin{vmatrix} 0.5 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{vmatrix}$$

All students have the same consistent model.

All students have a consistent model, but different ones.

Students have mixed models.

The state of the class may then be described by finding the eigenvalues and eigenvectors of the class density matrix.

Traditional statistical approaches that simple sum the scores over the class, effectively only retain the diagonal of the density matrix and therefore lose information about the mixing of student models. Traditional methods that retain the answer for each student in a large individual question response matrix (e.g. factor analysis) attempt to extract model information from the clustering of student responses. This only is effective if the students have consistent models. Consider for example a test of $m$ elements given to a class of $N$ students in which half the students answer consistently with model 1 and half the students answer consistently with model 2. A factor analysis will extract two strong factors. If, however, all of the students in the class are using both models, each one 50% of the time, the factor analysis will find no strong factor. The assumption of student consistency is built into the method and the information about student model mixing is lost. In our method, qualitative research is used to provide the extra information as to what models are common. This then allows the use of the matrix of student responses to extract information about the extent of student mixed model use.

## Example: FCI Measurement of the Force-Motion Model

As an example, we gave the FCI as pre and post tests to two sets of classes of students in the first semester of our calculus-based engineering course described above. (There was no laboratory in the first semester of the three-semester sequence.) One group (labeled *traditional*) was instructed with TA-led

recitation sections, the other (labeled *tutorial*) replaced these sections with the group-learning tutorial environment described above. Before instruction, the density matrix for both classes were dominated by a single eigenvector that was predominantly in the direction of model 1 (force is needed for motion). After instruction, the students in the traditional class were in a mixed state, with students switching frequently between models 1 and 2 (Newton 1 model). After tutorial instruction, the class was dominated by an eigenvector corresponding to the correct model. This is shown in figure 8 at the right. The figure shows the two-dimensional subspace of models 1 and 2. The points correspond to the eigenvectors. (The vectors themselves are not drawn to simplify the diagram. Each point should be thought of as a vector in the 1-2 space drawn starting from the origin). The distance of the point from the origin represents the fraction of students using this particular mixed model represented by the eigenvector. The angle of the eigenvector from the coordinate axes represents the amount of mixing. A figure of merit can be obtained by measuring the distance between the point corresponding to the pre-vector and the "ideal" point (0,1) (all students using the correct model all the time). The figure of merit is then the fraction of that distance achieved by the instructional method. We refer to this as the *instructional efficiency* (fraction of the possible gain). For these two methods, the tutorial class attained an instructional efficiency of 62% on the force-motion concept, while the traditional class achieved 29%.

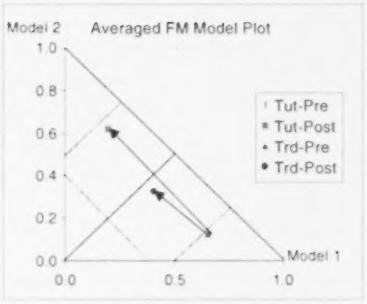


*Fig. 8:Development of student models of force-motion (dominant eigenvector of class density matrix) in response to two modes of instruction*

This method allows us to make a very detailed statement describing the state of the class before and after two different kinds of instruction and to compare them along a particular dimension of student understanding using the FCI as a probe. On the force-motion dimension, the traditional class moved from being dominated by a nearly pure state of a common alternative conception into a mixed (confused) state in which most students used both the correct and incorrect models. The tutorial class made a much stronger transition to a nearly pure state using the correct model.

The statements that we can make using our method provides a much more specific and useful description of the class than is possible using traditional methods such as factor analysis which cannot extract these details.[17]

## Conclusions

In this paper I have summarized the recent work of the Maryland Physics Education Research Group on analyzing student learning in introductory university physics. This work is carried out in a model of student thinking and learning basic on principles learned in cognitive science. The model permits us to better understand and diagnose the state of both individual students and classes. The most critical cognitive principles are:

- Student responses to a proposed physics context or problem are productive and context dependent.
- Student knowledge is organized into patterns of association or schema.

In the context of university physics, we conclude:

- The context that determines a student response includes not only the question or problem and the classroom environment, but unobservable (and uncontrollable) elements of the student's internal state. As a result, students can respond to questions that appear equivalent to an expert with

different, apparently contradictory models. This suggests that an appropriate way to think about student models is as if they were able to exist in different states at the same time.

- Tools developed by analogy with quantum theory (where quantum particles can exist in different states at the same time) allows us to describe the state of a class in more detail than traditional tools.

These approaches permit the diagnosing of the state of a class in detail. Having this information can help both with the creation and the evaluation of curricular innovations.

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[1] L. C. McDermott and E. F. Redish, "Resource Letter: PER-1: Physics Education Research," *Am. J. Phys.* **67**, 755-767 (1999).

[2] E. Mazur, *Peer Instruction: A User's Manual* (Prentice Hall, 1997).

[3] For example, I. Halloun and D. Hestenes, "Common sense concepts about motion," *Am. J. Phys.* **53**, 1056-1065 (1992).

[4] See for example, J. R. Anderson, *Rules of the Mind* (Lawrence Erlbaum, 1993) and P. S. Churchland and T. J. Sejnowski, *The Computational Brain* (MIT Press, 1993). For a review of cognitive science vis a vis physics teaching, see E. F. Redish, "Implications of cognitive studies for teaching physics," *Am. J. Phys.* **62**, 796-803 (1994).

[5] K. Wosilait, P. R. L. Heron, P. S. Shaffer, and L. C. McDermott, "Addressing student difficulties in applying a wave model to the interference and diffraction of light," *Physics Educ. Research, A Supplement to the Am. J. Phys.* **67:7 Supp. 1**, S5-S15 (1999)

[6] The term "schema" goes back to Emmanuel Kant and has been used by many authors in a variety of ways, having some common features but differing in detail. We use it here to refer to a moderately robust pattern of associations. The idea is not sharply defined (At what degree of strength do we stop using the term "schema" and go back to "pattern of association"?) but it really doesn't need to be for our purposes.

[7] M. Sabella, "Using the context of physics problem solving to evaluate the coherence of student knowledge", Ph.D. thesis, University of Maryland, 1999.

[8] M. Wittmann, "Making sense of how students come to an understanding of physics: An example from mechanical waves", Ph.D. thesis, University of Maryland, 1998.

[9] These are in the model of the University of Washington tutorials. See L. C. McDermott et al., *Tutorials in Introductory Physics* (Prentice Hall, 1998).

[10] E. J. Pedhazur and L. P. Schmelkin, *Measurement, Design, and Analysis: An Integrated Approach* (Lawrence Erlabuam, 1991).

[11] We use the abstract-vector bra/ket notation of quantum physics for a coordinate-free representation of a vector, $|u\rangle = (u_1, u_2, u_3, \ldots)^T$ and $\langle u| = (u_1, u_2, u_3, \ldots)$. ( The superscript "T" indicates transpose.)

[12] These models are described in the education community as "misconceptions" when the models are being denigrated and as "alternative conceptions" when the point is being emphasized that the productive elements within the student models may be resources that the instructor can use to help the student make the transition to a more complete and consistent model.

[13] L. Bao, "Model Analysis: A Quantitative Approach to Study Student Understanding of Physics", Ph.D. thesis, University of Maryland, 1999.

[14] D. Hestenes, M. Wells and G. Swackhamer, "Force Concept Inventory," *Phys. Teach.* **30**, 141-158 (1992).

[15] Note that the a correct model does not have to correspond to a correct answer. The student could be using the model incorrectly. For example, on the multiple-choice multiple-response wave question the student could answer that speeding up the pulse could be accomplished by decreasing the tension in the string. The answer is wrong, but the model that the properties of the medium determine the speed is correct.

[16] We should really include a specification of the student in the labeling of the number of answers given; thus, $n_1$, for example, should really be $n^k_1$. We suppress this extra index here for simplicity.

[17] D. Huffman and P. Heller, "What does the force concept inventory actually measure?" *Phys. Teach.* **33**, 138-143 (1995).